

Suppose two firms want to merge. In general, the cost function of a merged firm is given by

$$TC_m(Q) = \min_{q_A, q_B} [TC_A(q_A) + TC_B(q_B)], \quad \text{such that } q_A + q_B = Q \quad (1)$$

where TC_A and TC_B are potentially different cost functions for the pre-merger firms. Intuitively condition (1) makes sense, as the merged firm will try to reallocate resources among A and B so as to minimize the production costs. The Lagrangian associated with (1) is

$$\mathcal{L}(q_A, q_B; \lambda) = [TC_A(q_A) + TC_B(q_B)] + \lambda[Q - q_A - q_B]$$

If both firms are still in production after the merger, i.e. $q_A > 0$ and $q_B > 0$, then the FOC's for the Lagrangian will yield the following

$$\left. \begin{aligned} \frac{\partial TC_A(q_A)}{\partial q_A} = MC_A(q_A) = \lambda \\ \frac{\partial TC_B(q_B)}{\partial q_B} = MC_B(q_B) = \lambda \end{aligned} \right\} \Rightarrow MC_A(q_A) = MC_B(q_B)$$

This is how we get the condition that marginal costs should be equalized across the two firms.

Suppose A and B have constant but different marginal costs:

$$MC_A = a, \quad MC_B = b, \quad \text{where } b > a$$

Apparently, there is no way we can equate the two marginal costs. This corresponds to a case where there is a *corner solution* to problem (1), i.e. one of the two firms will be shut down after the merger (and naturally it's going to be firm B , whose MC is higher).¹

Formally solving for the optimal q_A and q_B , and showing that $q_A = Q$ and $q_B = 0$ require the use of the "Kuhn-Tucker Conditions", which is not in the scope of this course (you're likely to see it in ECON 331). Here we consider a specific example:

Suppose there is no fixed cost for either firm, and suppose $MC_A = 2$ and $MC_B = 3$. Then,

$$TC_A(q_A) = 2q_A, \quad TC_B(q_B) = 3q_B$$

To find the total cost of the merged firm, we apply (1):

$$\begin{aligned} TC_m(Q) &= \min_{q_A, q_B} [TC_A(q_A) + TC_B(q_B)], \quad Q = q_A + q_B \\ &= \min_{q_A, q_B} [2q_A + 3q_B] \end{aligned}$$

It is not hard to see that, to minimize the joint total costs ($TC_A + TC_B$) while keeping production level positive (i.e. $Q > 0$) implies that we should force $q_B = 0$ and let $q_A = Q > 0$. This conforms to the intuition that, when there is no capacity constraint in production, the merger between an efficient firm and an inefficient one should result in the shutdown of the inefficient firm.

¹ Note that here we're assuming implicitly that there are no capacity constraints in production.